Properties of Probability: If E is an event of the sample space S, then

1. $0 \leq P(E) \leq 1$
2. If $P(E) = 0$, then E is an impossible event.
3. $P(S) = 1$ Sum of all probabilities for an experiment is 1.

Probability Distribution - table showing all possible outcomes for an experiment and the corresponding probabilities.

Experiment: Rolling a Die

X (possible outcomes)	P(X)
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

Section 12.5 - Expected Value

expected value (mathematical expectation) -

a weighted average of the various probabilities of an experiment

the expected value deserves a label because it is a VALUE not a probability!

1. Suppose you roll a die and are paid $5 if it is a 3, 4, or 6, and $2 if it is a 1, 2, or 5. What is the expected value of this game?

Roll	Payout	P(x)
3, 4, or 6	$5	1/2
1, 2, or 5	$2	1/2

Expected value = sum of (payouts)(P(x))

$5\left(\frac{1}{2}\right) + 2\left(\frac{1}{2}\right)$

$2\frac{1}{2} + 1$

$3\frac{1}{2}$ $3.50

Every time you play, you can expect to earn $3.50.

2. Suppose that you roll two dice. You will be paid $5 if you roll a double. You will not receive anything for any other outcome. What is the expected value of this game?

Roll	Payout	P(x)
double	$5	$6/36 = 1/6$
not double	$0	$30/36 = 5/6$

$$\text{exp.val} = 5\left(\frac{1}{6}\right) + 0\left(\frac{5}{6}\right)$$

$$= \frac{5}{6}$$

$$= \$0.83$$

3. A box contains one each of $1, $5, $10, $20, and $100 bills. You reach in and withdraw one bill. What is the expected value?

Bill	Value	P(x)
	$1	1/5
	$5	1/5
	$10	1/5
	$20	1/5
	$100	1/5

$$1\left(\frac{1}{5}\right) + 5\left(\frac{1}{5}\right) + 10\left(\frac{1}{5}\right) + 20\left(\frac{1}{5}\right) + 100\left(\frac{1}{5}\right)$$

$$\frac{1}{5}(1 + 5 + 10 + 20 + 100)$$

$$\frac{1}{5}(136)$$

$$\boxed{\$27.20}$$

4. A punch-out card contains 100 spaces. One space pays $100, five spaces pay $10, and the others pay nothing. How much should you pay to punch out one space?

	Pay out	P(x)
	$100	1/100
	$10	5/100
	$0	94/100

$100\left(\frac{1}{100}\right) + 10\left(\frac{5}{100}\right) + 0\left(\frac{94}{100}\right)$

$1 + .5 + 0$

$\$1.50$

$1.50 or less

5. A game involves drawing a single card from an ordinary deck. If an ace is drawn, you receive $.50; if a face card is drawn is drawn, you receive $.25; if the two of spades is drawn, you receive $1. If the cost of playing is $.10, should you play?

card	payout amt	net earnings	P(x)
A	.50	.40	4/52
F	.25	.15	12/52
2 of S	1.00	.90	1/52
rest of cards	0	−.10	35/52

$.4\left(\frac{4}{52}\right) + .15\left(\frac{12}{52}\right) + .90\left(\frac{1}{52}\right) + -.10\left(\frac{35}{52}\right)$

$\frac{.4(4) + .15(12) + .9(1) - .1(35)}{52}$

$0.02

yes play the game

6. Suppose that you roll one die. You are paid $5 if you roll a one and you pay $1 otherwise. What is the expectation? Is this a fair game?

roll	payout	P(x)
1	$5	1/6
2,3,4,5,6	−$1	5/6

$5\left(\frac{1}{6}\right) - 1\left(\frac{5}{6}\right)$

$\frac{5}{6} - \frac{5}{6}$

$0 fair

7. Suppose you have 5 quarters, 5 dimes, 10 nickels, and 5 pennies in your pocket. You reach in and choose a coin at random so that you can tip your barber. What is the barber's expectation? What tip is the barber most likely to receive?

coin	value	P(x)
Q	.25	5/25
D	.10	5/25
N	.05	10/25
P	.01	5/25

exp = $0.09

most likely = $.05