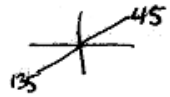
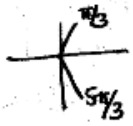
② $\sin 495$
-360
$\sin 135 = \boxed{\sin 45^\circ}$



③ $\cos \frac{23\pi}{3} - \frac{6\pi}{3} = \frac{17\pi}{3} - \frac{6\pi}{3} = \frac{11\pi}{3} - \frac{6\pi}{3} = \cos \frac{5\pi}{3} = \boxed{\cos \frac{\pi}{3}}$



④ $\cos \frac{13\pi}{12} = \cos\left(\frac{4\pi}{12} + \frac{9\pi}{12}\right)$

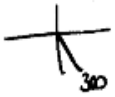
$= \cos\left(\frac{\pi}{3} + \frac{3\pi}{4}\right)$

$= \cos\frac{\pi}{3}\cos\frac{3\pi}{4} - \sin\frac{\pi}{3}\sin\frac{3\pi}{4}$

$= \left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \boxed{\frac{-\sqrt{2}-\sqrt{6}}{4}}$

⑤ $\tan 345 = \tan(300+45)$



$= \frac{\tan 300 + \tan 45}{1 - \tan 300 \tan 45}$

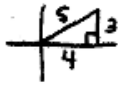
$= \frac{-\sqrt{3}+1}{(1+\sqrt{3})} \cdot \frac{1-\sqrt{3}}{(1-\sqrt{3})}$

$= \frac{1-2\sqrt{3}+3}{1-3}$

$= \frac{4-2\sqrt{3}}{-2}$

$\boxed{\tan 345 = -2+\sqrt{3}}$

⑥ $\sin\theta = \frac{3}{5}$



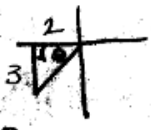
$\cos\theta = \frac{4}{5}$

$\sin 2\theta = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \boxed{\frac{24}{25}}$

$\cos 2\theta = \frac{16}{25} - \frac{9}{25} = \boxed{\frac{7}{25}}$

$\tan 2\theta = \frac{24}{25} \cdot \frac{25}{7} = \boxed{\frac{24}{7}}$

⑦ $\cot\theta = \frac{2}{3}$



$\tan\theta = \frac{3}{2}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$\frac{2\left(\frac{3}{2}\right)}{1-\left(\frac{3}{2}\right)^2}$

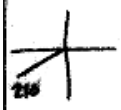
$\frac{3}{1-\frac{9}{4}}$

$\frac{12}{4-9}$

$\boxed{\tan 2\theta = \frac{12}{-5}}$

(8) $\sin 105 = \sin\left(\frac{1}{2} \cdot 210\right)$

$= \pm\sqrt{\frac{1-\cos 210}{2}}$



$= \pm\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} \cdot \frac{2}{2}$

$\sin 105 = \frac{\sqrt{2+\sqrt{3}}}{2}$

(9) $\sin 2x - 1 = 0$

$\sin^{-1}(\sin 2x) = (1)^{\sin^{-1}}$

$2x = \sin^{-1} 1$

$x = \frac{\sin^{-1} 1}{2}$

$x = \frac{90}{2}$

$x = 45^\circ$

(10) $2\cos^2 x - 2 = -3\cos x$

$2\cos^2 x + 3\cos x - 2 = 0$

$(2\cos x - 1)(\cos x + 2) = 0$

$2\cos x = 1$

$x = \cos^{-1}\frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

(11) $2\cos^2 x = 3\sin x$

$2(1-\sin^2 x) - 3\sin x = 0$

$2 - 2\sin^2 x - 3\sin x = 0$

$2\sin^2 x + 3\sin x - 2 = 0$

$(2\sin x - 1)(\sin x + 2) = 0$

$\sin x = \frac{1}{2}$

$x = \sin^{-1}\frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

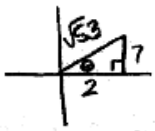
$x = \frac{\pi}{6} + 2n\pi$, n is integer

$x = \frac{5\pi}{6} + 2n\pi$, n is integer

(12) $\cot x = \frac{1}{4}$

$\tan x = 4$

(13)



$49+4 = 53$

$\csc x = \frac{1}{\sin x}$ $\sin x = \frac{7}{\sqrt{53}}$

$\csc x = \frac{\sqrt{53}}{7}$

(14) $\sin^2\theta\cos^2\theta + \cos^2\theta$

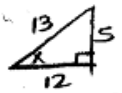
$\cos^2\theta(\sin^2\theta - 1)$

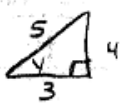
$-\cos^2\theta(1 - \sin^2\theta)$

$-\cos^2\theta(\cos^2\theta)$

$-\cos^4\theta$

(15)





$\sin x = \frac{5}{13}$ $\sin y = \frac{4}{5}$

$\cos x = \frac{12}{13}$ $\cos y = \frac{3}{5}$

$\cos(x+y) = \cos x\cos y - \sin x\sin y$

$\left(\frac{12}{13}\right)\left(\frac{3}{5}\right) - \left(\frac{5}{13}\right)\left(\frac{4}{5}\right)$

$= \frac{36}{65} - \frac{20}{65}$

$\cos(x+y) = \frac{16}{65}$

$$\frac{\csc x}{\cot x + \tan x} = \cos x$$

$$\frac{\frac{1}{\sin x} \cdot \sin x \cos x}{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x} \cdot \sin x \cos x}$$

$$\frac{\cos x}{\cos^2 x + \sin^2 x}$$

$$\frac{\cos x}{1}$$

$$\cos x \neq \cos x$$

(17) $$\tan x + \frac{\cos x}{1+\sin x} = \sec x$$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{1+\sin x} \qquad \frac{1}{\cos x}$$

$$\frac{\sin x(1+\sin x) + \cos^2 x}{\cos x(1+\sin x)}$$

$$\frac{\sin x + \sin^2 x + \cos^2 x}{\cos x(1+\sin x)}$$

$$\frac{\sin x + 1}{\cos x(1+\sin x)}$$

$$\frac{1}{\cos x} \neq \frac{1}{\cos x}$$

(18) $$1 + \sin 2x = (\sin x + \cos x)^2$$

$$1 + 2\sin x \cos x \mid \sin^2 x + 2\sin x \cos x + \cos^2 x$$

$$1 + 2\sin x \cos x = 1 + 2\sin x \cos x$$